

# Study of unsteady interactions between gases and solid particles

Progress report (FA4869-06-1-0042)

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14. ABSTRACT <b>For numerical analysis of shock wave propagation in gas-particle mixtures, drag coefficients of a sphere in steady flows are generally used. However, it is shown both experimentally and numerically that a shock loaded solid sphere experiences unsteady drag forces. The paper describes a model of unsteady drag force and its effect on the structure of the non-equilibrium region behind a shock front traveling in a dusty gas. The results are compared with those obtained by using a steady drag coefficient and are discussed. It is demonstrated that the large drag force at the early stage of the interaction between shock-wave induced flow and a solid particle affects the flow structure that is obtained with a steady drag force.</b>					
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# **1 Activities and results reported in the previous report - October 2006**

Two series of investigation were reported in the progress report of October 2006.

The first one was the study of spherical shock waves (blast waves) propagating in dusty gases and flow behind them. The purpose of the study was to see if we would observe the phenomena in which particles drive the gas. At the beginning when a spherically expanding shock wave is strong, solid particles that are originally stationary are accelerated by the gas behind the shock wave. Later when the shock wave expands further, it decelerates and becomes weaker. At this stage the particles have already gained certain amount of momentum from the gas behind the shock wave. Consequently, under certain conditions, it can be expected that particles now accelerate the gas, contrarily to the early stage of the explosion.

In conclusion, however, we did not observe the aforementioned phenomena at least within the range of parameters used in the current numerical analyses. The conclusion at the time of the previous report was that the range of parameters such as initial shock strength, particle diameters etc., was not wide enough. It was felt that more simulations were necessary to make any decisive conclusion.

In the second series of investigations, the non-equilibrium region behind a plane shock wave is studied by numerically integrating the conservation equations of mass, momentum and energy. Unsteady drag forces obtained both experimentally and numerically were modeled and implemented in the

numerical code. It is reported that the drag force is about an order of magnitude larger compared to the steady one when a shock wave passes over a sphere due to complex wave interactions. The purpose of the investigation was to evaluate the effect of unsteady drag force on the structure of non-equilibrium region behind a plane shock wave. The unsteady drag forces were obtained for stationary solid spheres of different diameters (Sun et al. 2004). In a dusty gas, a particle starts to move at the moment when a shock wave arrives at the particle. Simulations had been carried out for both cases of with and without the particle movement and the results were compared. This series of computations had not been completed in the previous report.

## 2 Progress report on the activities after the previous report

Following activities have been carried out since the previous progress report.

1. Investigations on the effect of unsteady drag force on the non-equilibrium region behind plane shock wave.
2. Development of three dimensional numerical code for simulating flows in a dusty gas.
3. Investigations of spherically expanding shock waves in dusty gases.

The item 1 is the continuation of the research activities described in the previous section. Computations of drag force for different incident shock Mach numbers were done to investigate the effect of incident shock Mach number on the unsteady drag force. The effect of different mass ratio between gas and particle phases was also investigated.

The results were presented at the 26th international shock wave symposium ("Effect of unsteady drag force on structure of non-equilibrium region behind shock wave in gas-particle mixture" by T. Saito, M. Saba, M. Sun and K. Takayama, Proc. 26th ISSW ). Also a paper entitled "The effect of an unsteady drag force on the structure of a non-equilibrium region behind a shock wave in a gas-particle mixture" is submitted to the international journal of Shock Waves and is accepted for publication.

The manuscripts of the proceedings of the ISSW26 and the manuscript submitted to the Shock Wave Journal are included in this progress report as. Both will be published soon.

The second item has practical purpose of applications to numerical simulations of several environmental problems such as volcano eruptions, dust explosions etc. with appropriate terrains into considerations. Investigations of generation and propagation of blast waves by explosive-type volcano eruptions have been carried out in our group for many years. We expect that the development of the 3D numerical code will provide us more realistic information to study the mechanism of volcano eruptions and also to establish better safety measures.

The third item is a continuation of the work that is described in the previous section. Due to limited range of computational parameter, we did not observe the phenomenon that particles drive the gas. However, we felt that there is much to be studied on behavior of blast waves in dusty gases. Simulations with wider range of computational parameters are now being considered and simulations are being carried out.

### 3 Development of 3D numerical code

Three-dimensional numerical code is developed for simulating shock wave propagation in dusty gases. The code just started to run and we are testing the code by applying it to a couple of simple test problems.

The code is briefly described in this section by showing the basic equations that are solved numerically. Numerical results of test problems are also presented.

#### 3.1 Basic equations and numerical method

The governing equations of the dusty gas flows in three spatial dimensions (3D) can be written as:

$$\frac{\partial U_g}{\partial t} + \frac{\partial F_g}{\partial x} + \frac{\partial G_g}{\partial y} + \frac{\partial H_g}{\partial z} = -I, \quad (1)$$

$$\frac{\partial U_d}{\partial t} + \frac{\partial F_d}{\partial x} + \frac{\partial G_d}{\partial y} + \frac{\partial H_d}{\partial z} = I. \quad (2)$$

here, the vectors of conserved quantities and fluxes for the gas are

$$U_g = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{pmatrix}, F_g = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ E \end{pmatrix}, G_g = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ (E + p)v \end{pmatrix}, H_g = \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ (E + p)w \end{pmatrix}, \quad (3)$$

and for solid particles

$$U_d = \begin{pmatrix} \sigma \\ \sigma u_d \\ \sigma v_d \\ \sigma w_d \\ \Omega \end{pmatrix}, F_d = \begin{pmatrix} \sigma u_d \\ \sigma u_d^2 \\ \sigma u_d v_d \\ \sigma u_d w_d \\ \Omega u_d \end{pmatrix}, G_d = \begin{pmatrix} \sigma v_d \\ \sigma u_d v_d \\ \sigma v_d^2 \\ \sigma v_d w_d \\ \Omega v_d \end{pmatrix}, H_d = \begin{pmatrix} \sigma w_d \\ \sigma u_d w_d \\ \sigma v_d w_d \\ \sigma w_d^2 + p \\ \Omega w_d \end{pmatrix}. \quad (4)$$

The vector of interaction terms is

$$I = \frac{\sigma}{m} \begin{pmatrix} 0 \\ D_x \\ D_y \\ D_z \\ Q + u_d D_x + v_d D_y + w_d D_z \end{pmatrix}. \quad (5)$$

The total energies of gas and particles are

$$E = \rho \left\{ C_v T + \frac{1}{2} (u^2 + v^2 + w^2) \right\}, \quad (6)$$

$$\Omega = \sigma \left\{ C_m \Theta + \frac{1}{2} (u_d^2 + v_d^2 + w_d^2) \right\}, \quad (7)$$

and here  $u$  and  $u_d$  are the x-component of gas and dust velocity and similarly  $v$  and  $v_d$  for the y-component. The x and y-component of the drag force are given as

$$D_x = \frac{1}{8} \pi d^2 \rho (u - u_d) |u - u_d| C_D, \quad (8)$$

$$D_y = \frac{1}{8} \pi d^2 \rho (v - v_d) |v - v_d| C_D. \quad (9)$$

$$D_z = \frac{1}{8} \pi d^2 \rho (w - w_d) |w - w_d| C_D. \quad (10)$$

Here as in the 1D case, the following models of the drag coefficient and the Nusselt number are assumed.

$$C_D = 0.48 + 28 \text{Re}^{-0.85}, \quad (11)$$

$$\text{Nu} = 2.0 + 0.6 \text{Pr}^{\frac{1}{3}} \text{Re}^{\frac{1}{2}}, \quad (12)$$

The partical Reynolds number is defined in 2D as follows,

$$\text{Re} = \frac{\rho d \sqrt{(u - u_d)^2 + (v - v_d)^2 + (w - w_d)^2}}{\mu}. \quad (13)$$

As to the solution strategy, the operator splitting is used for handling the interaction terms. The homogeneous part of the governing equations (14)



and (15) below are solved first and then the solution is further modified by taking the effect of interaction between the gas and solid particle phases by solving (16) and (17).

$$\frac{\partial U_g}{\partial t} + \frac{\partial F_g}{\partial x} + \frac{\partial G_g}{\partial y} + \frac{\partial H_g}{\partial z} = O, \quad (14)$$

$$\frac{\partial U_d}{\partial t} + \frac{\partial F_d}{\partial x} + \frac{\partial G_d}{\partial y} + \frac{\partial H_d}{\partial z} = O. \quad (15)$$

$$\frac{dU_g}{dt} = -I, \quad (16)$$

$$\frac{dU_d}{dt} = I. \quad (17)$$

In solving (14) and (15), we had options to use Strang type operator splitting to treat the multi space dimensions (Strang, G. (1968): On the construction and comparison of difference schemes, SIAM J. Numer. Anal. 5(3):506-517). In this study, however, the numerical fluxes in both space dimensions are considered at once.

For the evaluation of numerical fluxes of the particle phase, flow parameters are interpolated between neighboring computational cells as follows,

$$q(s, 0) = \frac{\Delta q}{\Delta s} s + q^\circ, \quad (18)$$

here  $q^\circ$  is the initial value at the cell interface,

$$q^\circ = \frac{dr \cdot q_l + dl \cdot q_r}{\Delta s}, \quad \Delta s = dl + dr. \quad (19)$$

The interpolated values of flow parameters at the cell interfaces are ob-

tained from the following equations

$$u_d = \frac{1}{1 + \left(\frac{\Delta u_d}{\Delta s}\right)_t} u_d^\circ, \quad (20)$$

$$v_d = \frac{1}{1 + \left(\frac{\Delta u_d}{\Delta s}\right)_t} \left[ \left\{ \left(\frac{\Delta u_d}{\Delta s}\right) v_d^\circ - \left(\frac{\Delta v_d}{\Delta s}\right) u_d^\circ \right\} t + v_d^\circ \right], \quad (21)$$

$$w_d = \frac{1}{1 + \left(\frac{\Delta u_d}{\Delta s}\right)_t} \left[ \left\{ \left(\frac{\Delta u_d}{\Delta s}\right) w_d^\circ - \left(\frac{\Delta w_d}{\Delta s}\right) u_d^\circ \right\} t + w_d^\circ \right], \quad (22)$$

$$\Theta = \frac{1}{1 + \left(\frac{\Delta u_d}{\Delta s}\right)_t} \left[ \left\{ \left(\frac{\Delta u_d}{\Delta s}\right) \Theta^\circ - \left(\frac{\Delta \Theta}{\Delta s}\right) u_d^\circ \right\} t + \Theta^\circ \right], \quad (23)$$

$$\sigma = \frac{1}{\left\{ 1 + \left(\frac{\Delta u_d}{\Delta s}\right)_t \right\}^2} \left[ \left\{ \left(\frac{\Delta u_d}{\Delta s}\right) \sigma^\circ - \left(\frac{\Delta \sigma}{\Delta s}\right) u_d^\circ \right\} t + \sigma^\circ \right]. \quad (24)$$

The numerical fluxes at cell interfaces are calculated from the values with  $t = \frac{1}{2}\Delta t$ . Full details of the computational methods for 2D are available in the reference (Saito, T., Marumoto, M., Takayama, K.(2003): Numerical investigations of shock waves in gas-particle mixtures. Shock Waves, No.13, 299-322).

## 3.2 Preliminary test calculations

Two problems were calculated for testing the numerical code. Although, in principle, the code is simply an extension of our 2D code, it is significantly more complex. The code is developed on general curvilinear coordinates and runs in parallel on multiple processing elements by using MPI libraries.

### 3.2.1 Explosion of a pressurized cube under a dusty gas layer

The initial state of the problem is presented in Fig. 1. A cubic container filled with high pressure gas without dust particles is placed on the ground. The region between the two gray horizontal planes is filled with a dusty gas. The container is broken at time zero generating a blast wave. The blast wave interacts with the dusty-gas layer and the state of the mass concentration at

some time later is displayed in Fig. 2. It is seen that the borders between pure gas and the dusty gas are deformed by the blast wave and the flow behind it.

Since this is only a test for checking the code performance, not much to discuss from the physical point of view. However, the three dimensional nature of the flow is confirmed. Also we can see that the parallel execution of the program is performed nicely, since there is no obvious disturbances observed near the border of the regions calculated with different processors.

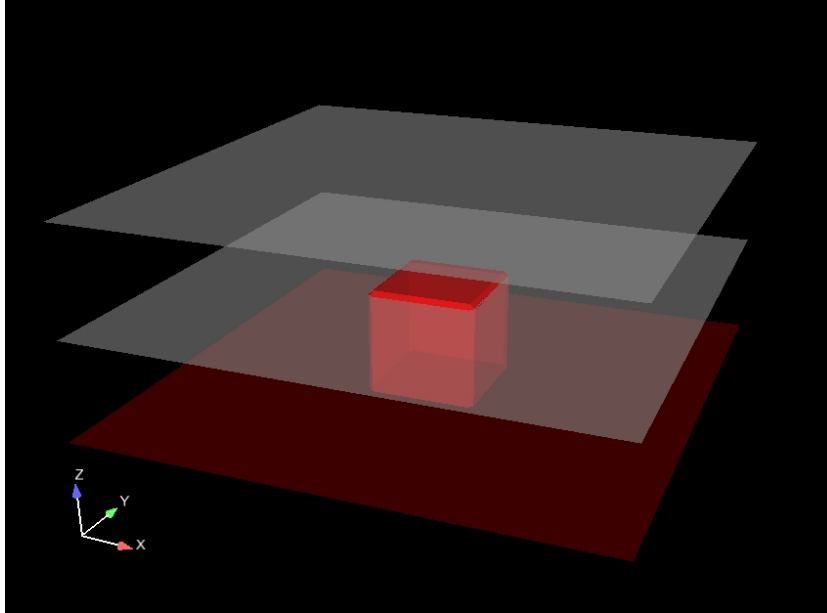


Figure 1: Initial state of the test problem,  $\tau = 0.0$ .

### 3.2.2 Explosion of a high pressure dusty gas in a cubic container

In contrast to the previous problem, a high pressure dusty gas is exploded into pure gas. Figure 3 presents the initial condition of the problem.

After the instantaneous burst of the cubic container that is filled with a high pressure dusty gas at some distance above the ground, blast wave is

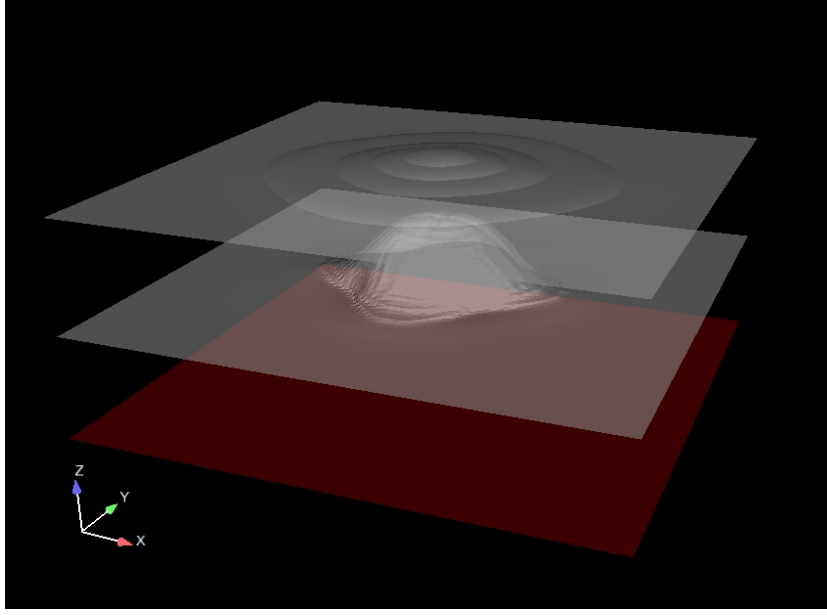


Figure 2: Distribution of particle mass concentration,  $\tau = 20.0$ .

generated and propagates into the surrounding pure gas. The distribution of particle mass concentration at a normalized time ( $\tau = 20.0$ ) is shown in Fig. 4. Complex distribution pattern of solid particles from the container is observed.

In conclusion, it seems that the 3D numerical code is developed and running properly.

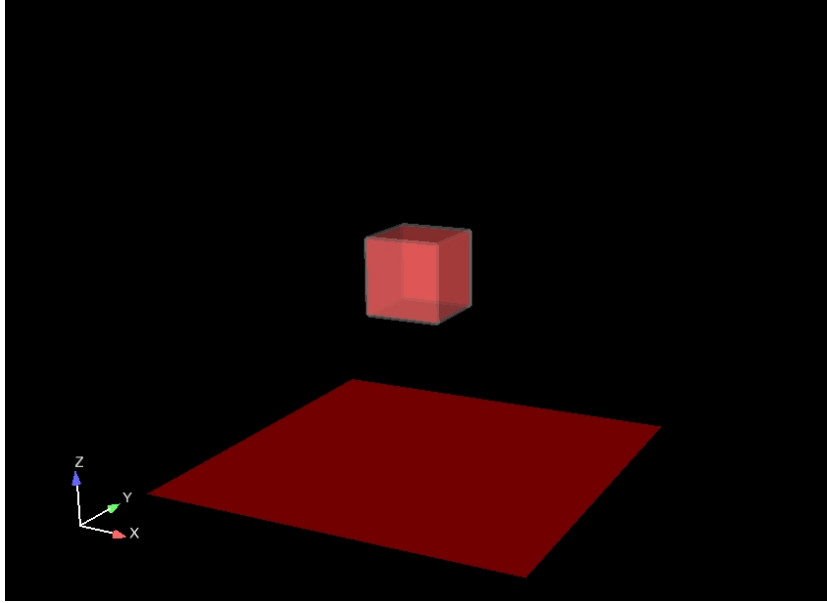


Figure 3: Initial state of the explosion of a cubic container filled with dusty gas,  $\tau = 0.0$ .

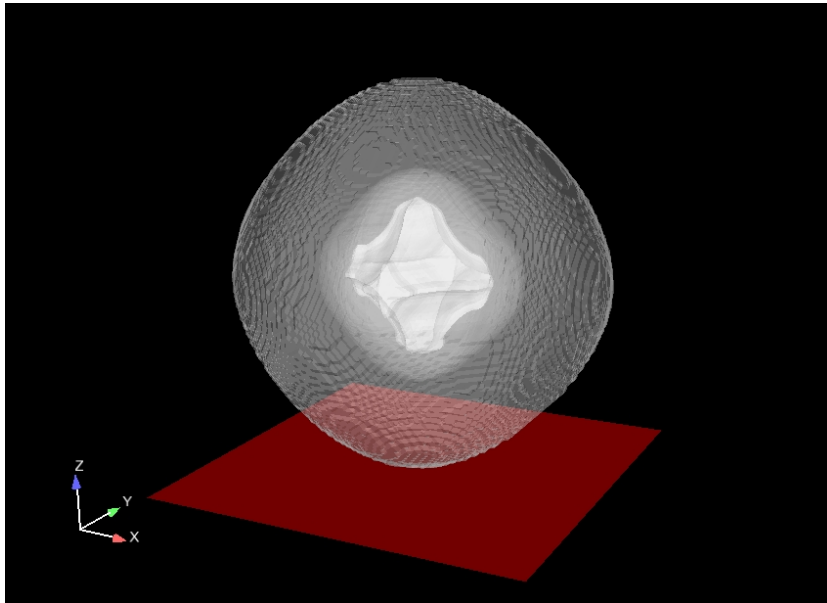


Figure 4: Distribution of particle mass concentration at values of 0.0001, 0.001 and 0.2,  $\tau = 20.0$ .

## 4 Future work

We plan to carry out following research subjects.

1. Numerical investigations of spherical shock waves in dusty gases.
2. Applications of 2D and 3D numerical codes to practical problems.

As a continuing research project, we investigate spherically expanding shock waves in dusty gases. We analyze the numerical results that have been accumulated since the beginning of this research project.

One of the candidates for application of the 2D code is the simulation of the starting and stopping of supersonic dusty gas flow in a nozzle. The subject is investigated for pure gases but not much is reported for cases with dusty gases.

As an application of the 3D numerical code, we will simulate imaginary eruption of a volcano. With the 3D dusty gas code, it is expected that we will be able to obtain some useful information regarding pyrocrastic flows. We have detailed information about Mount Usu, one of active volcanoes in Hokkaido Japan, together with high resolution digital elevation map of the area.